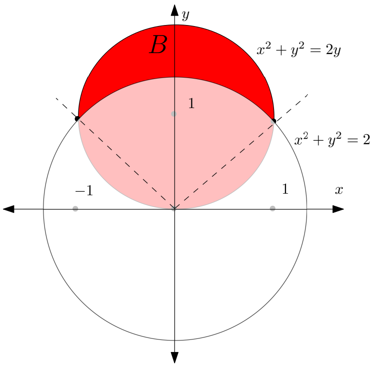
1. 

$B:\ -1 \le x \le 1,\ \sqrt{2-x^2} \le y \le 1+\sqrt{1-x^2}$

(I. Tip) Bölgesi için, (soldaki şekil) Fubini nin Teoreminden,

$$\int_{-1}^{1}\left(\int_{\sqrt{2-x^2}}^{1+\sqrt{1-x^2}} \sqrt{x^2+y^2}\,dy\right)dx = \int_B \sqrt{x^2+y^2}\,dA \text{ olur.}$$

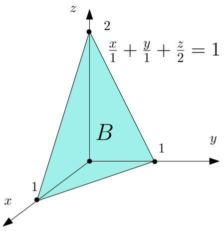
Kutupsal Koordinatlarda:

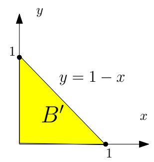
$B: \frac{\pi}{4} \le \theta \le \frac{3\pi}{4}, \quad \sqrt{2} \le r \le 2\sin\theta$ olarak yazılabilir.

2 Katlı İntegraller için Değişken Değiştirme Teoreminden,

$$\int_B \sqrt{x^2+y^2}\,dA = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\int_{\sqrt{2}}^{2\sin\theta} r\,r\,dr\,d\theta = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\int_{\sqrt{2}}^{2\sin\theta} r^2\,dr\,d\theta$$

$$= \frac{1}{3}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\left(8\sin^3\theta - 2\sqrt{2}\right)d\theta = \frac{40-6\pi}{9\sqrt{2}}$$

2. 



$\bar{y} = \dfrac{\int_B yx\,dV}{\int_B x\,dV}$ dir.

Bölgeyi (B' : xy-düzlemine izdüşümü olmak üzere)

$B = \{(x,y,z) : (x,y) \in B',\ 0 \le z \le 2(1-x-y)\}$

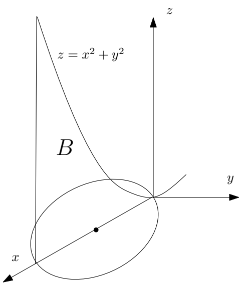
şeklinde yazabiliriz. Fubini (tipi) Teoreminden,

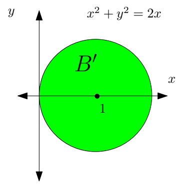
$$\int_B x\,dV = \int_{B'}\left(\int_0^{2-2x-2y} x\,dz\right)dA, \quad \int_B yx\,dV = \int_{B'}\left(\int_0^{2-2x-2y} xy\,dz\right)dA$$

olur. $B' : 0 \le x \le 1,\ 0 \le y \le 1-x$ olup:

$$\int_B x\,dV = \int_0^1\int_0^{1-x} x(2-2x-2y)\,dy\,dx = \frac{1}{12}$$

$$\int_B xy\,dV = \int_0^1\int_0^{1-x} xy(2-2x-2y)\,dy\,dx = \frac{1}{60}, \quad \text{ve } \bar{y} = \frac{1}{5} \text{ olur.}$$

3. 



Kütle$=\int_B \sqrt{x^2+y^2}\,dV$ dir.

$B = \{(x,y,z) : (x,y) \in B',\ 0 \le z \le x^2+y^2\}$

(B', B nin xy-düzlemine izdüşümü) olur.

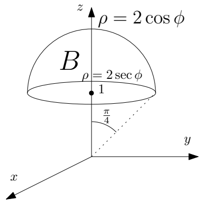
Bu bölge üzerinde integral hesaplamak için silindirik koordinatlar (eşdeğer olarak, B' üzerinde integral için kutupsal koordinatlar) kullanmak uygundur.

$B' : -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}, \quad 0 \le r \le 2\cos\theta$ olup,

(Silindirik Koordinatlarda) $B : -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}, \quad 0 \le r \le 2\cos\theta,\ 0 \le z \le r^2$ olur.

$$\text{Kütle} = \int_B \sqrt{x^2+y^2}\,dV = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\cos\theta}\int_0^{r^2} r\,r\,dz\,dr\,d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\cos\theta}\int_0^{r^2} r^2\,dz\,dr\,d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\cos\theta} r^4\,dr\,d\theta = \frac{32}{5}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^5\theta\,d\theta = \frac{64}{5}\int_0^{\frac{\pi}{2}}\cos^5\theta\,d\theta = \frac{512}{75}$$

4. 

Kütle$=\int_B \mu\, dV$. Bölgenin ve yoğunluğun z eksenine göre simetrik olması nedeniyle, kütle merkezi z-ekseni üzerindedir. Bu da $\bar{x} = \bar{y} = 0$ olması demektir. $\bar{z} = \frac{\int_B z\mu\, dV}{\int_B \mu\, dV}$ dır. Küresel Koordinatlarda:
$B : 0 \leq \theta \leq 2\pi,\ 0 \leq \phi \leq \frac{\pi}{4},\ \sec\phi \leq \rho \leq 2\cos\phi,\ \mu = \rho^2$ olur.

3-katlı integrallerde Değişken Değiştirme Teoreminden,

$$\int_B \mu\, dV = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_{\sec\phi}^{2\cos\phi} \rho^2\, \rho^2 \sin\phi\ d\rho\, d\phi\, d\theta = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_{\sec\phi}^{2\cos\phi} \rho^4 \sin\phi\ d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{4}} \frac{1}{5}(32\cos^5\phi - \sec^5\phi)\sin\phi\, d\phi\, d\theta$$

$$= \frac{2\pi}{5}\left(-\frac{32}{6}\cos^6\phi - \frac{1}{4}\sec^4\phi\right)\Big|_0^{\frac{\pi}{4}} = \frac{47\pi}{30}$$

$$\int_B z\mu\, dV = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_{\sec\phi}^{2\cos\phi} \rho\cos\phi\, \rho^2\, \rho^2 \sin\phi\ d\rho\, d\phi\, d\theta = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_{\sec\phi}^{2\cos\phi} \rho^5 \cos\phi\ \sin\phi\ d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{4}} \frac{1}{6}(64\cos^6\phi - \sec^6\phi)\sin\phi\, d\phi\, d\theta$$

$$= \frac{\pi}{3}\left(-\frac{64}{7}\cos^7\phi - \frac{1}{5}\sec^5\phi\right)\Big|_0^{\frac{\pi}{4}} = \frac{(109 - 16\sqrt{2})\pi}{35}$$

Bunlardan $\bar{z} = \dfrac{654 - 96\sqrt{2}}{329}$ elde edilir.